Extensive Form Games

Tutorial 3 Solutions

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1 Question 1

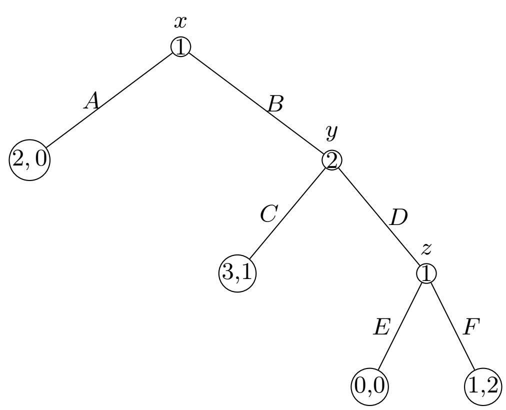

Figure 1: The game in extensive form.

Part b Player I, has 4 pure strategies. Player II has 2 pure strategies.

Part c 3 subgames at sub-trees rooted under nodes x, y, z. I'll accept two as an answer as well, but the whole tree is a sub-game of itself.

Part d

Player I \ Player II	C	D
AE	2, 0	2, 0
AF	2, 0	2, 0
BE	3,1	0, 0
BF	3, 1	1,2

NE : (AE, D), (BE, C), (AF, D) All the boxes with both numbers coloured are best responses for both players.

Zermelo Backward Prop - (AF, D)

2 Question 2

For all vertices in a specific information set, they must have the same available actions. Thus, for a given information set, there can be at most m strategies. There are k information sets, so the total number of pure actions is m^k

If each information set had had m_j pure actions, and there were k information sets in total for that user. Then the total number of pure actions is $\prod_{i=1}^{K} m_i$.

3 Question 3

Part a For each of the games, what information can player I lose while playing the game?

a. As player I cannot tell if player II plays D or E, it must be the case that player I also does not remember if they played A or B. If this is the case, it cannot remember player Two's actions at all, even if they played C or F.

b. Player I forgets if it's their first turn or second turn. Thus, it has no idea what player II did.

c. Player I has forgotten the whole game. It cannot remember any action by any player.

Part b For each game, write down the information sets and pure strategies for both players.

Part a

$I_1^{(1)} = \{x_1\}, \quad I_2^{(1)} = \{x_2, x_3\},$
$I_1^{(2)} = \{y_1\}, \quad I_2^{(2)} = \{y_2\}.$

$\Sigma_1 = \{A, B\} \times \{G, H\}$
$\Sigma_2 = \{C, D\} \times \{E, F\}$

Part b

$I_1^{(1)} = \{x_1, x_2\},$
$I_1^{(2)} = \{y_1\}$

$\Sigma_1 = \{A, B\},$
$\Sigma_2 = \{C, D\}$

Part c

$I_1^{(1)} = \{x_1\}, \quad I_2^{(1)} = \{x_2\}, \quad I_3^{(1)} = \{x_3, x_4\},$
$I_1^{(2)} = \{y_1\}, \quad I_2^{(2)} = \{y_2\}, \quad I_3^{(2)} = \{y_3\}.$

$\Sigma_1 = \{C, D\} \times \{E, F\} \times \{K, L\}$
$\Sigma_2 = \{A, B\} \times \{I, J\} \times \{G, H\}$

Part c For game B, identify any pure strategy Nash equilibria.

		Player II	
		C	D
Player I	A	7, 8	7, 8
	B	5, 0	1, 2

So (A, C) and (A, D) are in NE.

4 Question 4

Recall the forgetful driver example from the lecture: a 1-player "game" of imperfect recall. A behavioural strategy for this game can be defined as a probability p that the driver will exit (and so will drive straight with probability $1 - p$.

Part A Write down the expression for the expected utility a driver will obtain from playing a behavioural strategy p.

$$\begin{aligned} u(p) &= 0 \cdot p + (1-p)\,(4p + (1-p) \cdot 1) \\ &= -3p^2 + 2p + 1 \end{aligned}$$

Part B Use the expression you derived in the first part of this question to compute a value for p that maximises expected utility for the driver.

Taking the derivative with respect to p and setting it to 0, we get that the above equation is maximised[1] at $p^* = \frac{1}{3}$, to get utility $\frac{4}{3}$.

5 Question 5

Part A Informally explain what TAT-FOR-TIT does.

The player starts at defects and stays there as long as the other player cooperates. If the other player defects, then TAT FOR TIT asks the player to always do the opposite of what it was doing. It will stay at this new decision, as long as the other player cooperates.

Part B Consider each strategy playing against each other strategy (including itself). Compute the runs that would be generated, and identify the finite but in-finitely repeating sequence of outcomes. Use this repeating sequence to compute the utility obtained by each strategy in each pairing.

GRIM vs GRIM

	0	1	2	3	4	...	
GRIM	C	C	C	C	C	...	utility=-1
GRIM	C	C	C	C	C	...	utility=-1

GRIM vs TIT FOR TAT

	0	1	2	3	4	...	
GRIM	C	C	C	C	C	...	utility=-1
TIT-FOR-TAT	C	C	C	C	C	...	utility=-1

GRIM vs TAT FOR TIT

	0	1	2	3	4	...	
GRIM	C	D	D	D	D	...	utility -1
TAT FOR TIT	D	D	C	D	C	...	utility = -2.5

The repeating sequence for player 1 is DDDD... and for player 2 DCDC...

[1] Make sure to also check extremal conditions.

TAT FOR TIT vs TAT FOR TIT

	0	1	2	3	4	...	
TAT FOR TIT	D	C	C	C	C	...	utility -1
TAT FOR TIT	D	C	C	C	C	...	utility = -1

TAT FOR TIT vs TIT FOR TAT

	0	1	2	3	4	...	
TAT FOR TIT	D	D	C	D	D	...	utility $\frac{-5}{3}$
TIT FOR TAT	C	D	D	C	C	...	utility = $\frac{-5}{3}$

TIT FOR TAT vs TIT FOR TAT

	0	1	2	3	4	...	
TIT FOR TAT	C	C	C	C	C	...	utility -1
TIT FOR TAT	C	C	C	C	C	...	utility = -1

Part C Which of these pairs of strategies do you think forms a Nash equilibrium? (An informal argument will suffice.)

		Player II		
		Grim	Tat	Tit
Player I	Grim	-1,-1	-1,-2.5	-1,-1
	Tat	-2.5, -1	-1,-1	$\frac{-5}{3}$, $\frac{-5}{3}$
	Tit	-1, -1	$\frac{-5}{3}$, $\frac{-5}{3}$	-1,-1

Theorem 1 (Nash Folk) In an infinitely repeated game, every outcome in which every player gets at least their security value can be sustained as a Nash equilibrium.

$$\bar{u}_i\,(\text{Grim}) = \max\{-1, -1, -1\} = -1$$
$$\bar{u}_i\,(\text{Tat}) = \max\{-2.5, -1, -5/3\} = -1$$
$$\bar{u}_i\,(\text{Grim}) = \max\{-1, -5/3, -1\} = -1$$

As the values are all equal, then the security of a player $i \in \{1, 2\}$ is -1. Thus, all outcomes that allow (-1,-1) utility in the table above are in the NE.

6 Problem 6

Part A Suppose one of these machines has both statesl abelled with the same action (we have either C in both ovals, or D in both ovals). What behaviour does it generate? Can you simplify the automaton at all?

If both states have C, it means regardless of what the other players actions are, the player will always remain in the same state. This is the same ALLC in the lecture slides. Symmetrical argument with D.

Part B All the possible drawings

Here's how you draw them. For a given state, let 0 indicate the self loop arc, and 1 indicate the cross-over arc. These are the only two possible state transitions I can have in a 2 state automaton.

The other player has two actions, C and D. There are 4 possible labellings. $00, 01, 10, 11$ which is the same as saying both C, D take the self loop, C takes the self loop and D takes the cross-over arc, and so on. Similarly in the other state we have 4 choices as well. But two have two states guaranteed, we cannot allow the 00 option for the other state as well. That gives us 3 possible configurations for second state $01, 10, 11$ which is saying C takes self and D takes cross, C takes cross and D takes self, both C, D take cross. This gives us 4×3 configurations, plus the single state. That's 13 configs. Now re-label the two states, and we get 13 times $2 = 26$ configs.